ELECTRODYNAMIC FLOWS OF MEDIA OBEYING A NONLINEAR RHEOLOGICAL LAW

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UDC 532.135

Examples of the one-dimensional, steady-state electrodynamic flow of liquids characterized by power-type rheological laws and Shvedov–Bingham plastics in channels with dielectric walls in a uniform longitudinal electric field are considered.

The flow of liquids and gases containing three-dimensional distributions of electric-charge density, subject to the influence of an external electric field (electrohydrodynamic or EHD flows [1-7]), is now being intensively studied. In the chemical and petrochemical industries, in polymer technology, and also in connection with specific biomechanical problems, interest is centered in the EHD flow of media possessing mechanical properties more complicated than viscous Newtonian liquids. As the mathematical model of a continuous medium, describing the flows of various emulsions, suspensions, and similar finely-dispersed media, we may take the well-known three-constant rheological model

$$\tau = \tau_0 \left| \frac{dU}{dx} \right|^{-1} \frac{dU}{dx} + \eta \left| \frac{dU}{dx} \right|^{n-1} \frac{dU}{dx}; \ |\tau| > \tau_0. \tag{1}$$

As $\tau_0 \to 0$, Eq. (1) describes a liquid with a power-type rheological law, while for n = 1 it describes a viscoplastic medium or Shvedov–Bingham plastic. Examples of the one-dimensional steady-state gradient and shear flow of this kind of medium in a plane channel, a round tube, and a coaxial system may be found in [8-12].

In individual cases a finely dispersed emulsion or suspension may contain electrically charged particles. The existence of a three-dimensional electric-charge density may be accompanied by considerable changes in the velocity distribution of the medium when a longitudinal external electric field is applied to the latter. An example of a medium with properties similar to those of the model under consideration is a suspension of diatomite in transformer oil [13], comprising 10% diatomite, 89% transformer oil, 0.7% activator (water), and 0.3% oleic acid. An experimental investigation based on an electroviscometer led to the conclusion [13] that, in the rheological respect, dielectric flowing media with finely dispersed structures behaved as typical pseudoplastics in external electric fields. We also note that in a number of investigations relating to the mechanical properties of finely dispersed media the existence of a limiting tangential shear stress in the latter was indicated in [14]. In view of all this the applicability of the rheological model in form (1) to the study of EHD flow in finely dispersed media may be regarded as reasonably well-based.

In this paper we shall consider some examples of the EHD flows of liquids with a power-type rheological law (τ_0 = 0) in a plane channel, and those of a viscoplastic medium (n = 1) in a round tube with dielectric walls.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 27, No. 5, pp. 850-859, November, 1974. Original article submitted February 11, 1974.

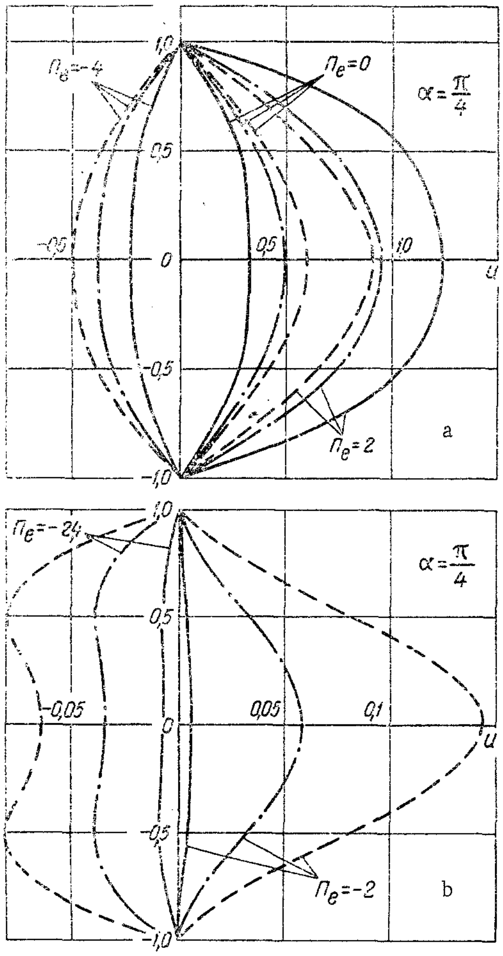

Fig. 1. Velocity distribution of a power-law liquid in a plane channel for positive and relatively large negative values of the parameter Π_e (a), and for small negative values of the parameter Π_e (b) (continuous curves n = 0.5; broken curves 1.5; dotted and dashed curves 1.0). The vertical axis represents x.

1. Gradient Flow of a Power-Law Liquid in a Plane Channel. Let an incompressible liquid with a rheological law of form (1), having $\tau_0 = 0$, flow in a plane channel with dielectric walls ($-a \leq x \leq a$) in the presence of a uniform external electric field $E_z = E_0 = \text{const}$. We shall assume that the liquid contains a three-dimensional distribution of electric-charge density $\rho = \rho(x)$ in which the charge per unit length of channel

$q_0 = \int_{-a}^{a} \rho(x)\,dx$ is one of the defining parameters of the problem. As characteristic quantities we take the half-width of the channel a, the charge density $\rho_0 = q_0/a$, and the electric field $e_0 = q_0/(\varepsilon\varepsilon_0)$. Under these conditions (according to [2]) the distribution of dimensionless charge density and the intensity of the induced electric field take the form

$$\rho(x) = \frac{\alpha}{2\,\mathrm{tg}\,\alpha} \cdot \frac{1}{\cos^2 \alpha x}, \qquad e(x) = \frac{1}{2}\,\frac{\mathrm{tg}\,\alpha x}{\mathrm{tg}\,\alpha}, \tag{1.1}$$

where the quantity $\alpha < \pi/2$ is defined by the equation

$$\alpha\,\mathrm{tg}\,\alpha = \frac{bq_0 a}{4D\varepsilon\varepsilon_0}. \tag{1.2}$$

Let us consider flow in a channel under the action of a pressure gradient $P = -(\partial p/\partial z) > 0$ constant in time. Taking the quantity $U_0 = (a^{n+1}P/\eta)^{1/n}$ as the characteristic velocity of the problem, we may write the dimensionless equation of motion of the medium and the boundary conditions of the problem for the flow under consideration thus

$$(|U'|^{n-1}U')' + \Pi_e \rho(x) = -1, \qquad U(\pm 1) = 0. \tag{1.3}$$

Here and subsequently the prime denotes a derivative with respect to the coordinate x. The parameter characterizes the ratio of the electric forces to the pressure forces. We note that the sign of Π_e depends on the direction of the external electric field ($E_0 \gtrless 0$ for $\Pi_e \gtrless 0$). Integrating Eq. (1.3) with due allowance for the flow symmetry condition $U'(0) = 0$ we obtain

$$|U'|^{n-1}U' = f_1(x) = -x - \frac{\Pi_e}{2}\,\frac{\mathrm{tg}\,\alpha x}{\mathrm{tg}\,\alpha}. \tag{1.4}$$

Subsequently we shall confine attention to consideration of the flow region $0 \leq x \leq 1$. Allowing for the presence of the modulus of the derivative $U'(x)$ in Eq. (1.4), let us study the sign of the latter; for this purpose we must distinguish three modes of flow, depending on the size of the parameter Π_e.

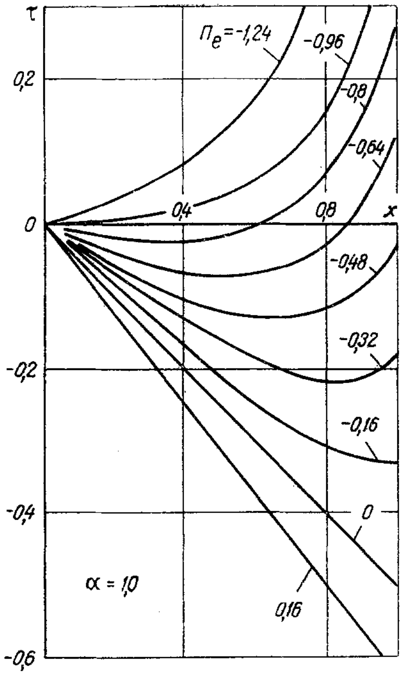

Fig. 2. Distribution of tangential shear stresses in a round tube.

a) If $\Pi_e > \Pi_1 = -2$, then $U'(x) < 0$ in the range $0 < x < 1$ and only equals zero in the center of the channel; the rate of liquid flow is positive, and on allowing for (1.4) and the condition of adhering to the wall of the channel $U(1) = 0$ it may be written in the form of a quadrature

$$U(x) = \int_x^1 |f_1(x)|^{\frac{1}{n}} dx = \int_x^1 \left(x + \frac{\Pi_e}{2} \frac{\operatorname{tg} \alpha x}{\operatorname{tg} \alpha}\right)^{\frac{1}{n}} dx. \quad (1.5)$$

b) If $\pi_e < \pi_2 = -2 \tan\alpha/\alpha$, we have $U'(x) > 0$ for $0 < x < 1$, U' being equal to zero in the center of the channel; the rate of liquid flow is negative:

$$U(x) = \int_1^x [f_1(x)]^{\frac{1}{n}} dx. \quad (1.6)$$

c) If $\Pi_2 < \Pi_e < \Pi_1$, there is a point x_0 inside the channel at which the derivative $U'(x)$ changes sign; for $x_0 < x < 1$ we have $U'(x) > 0$ and for $0 < x < x_0$ — $U'(x) < 0$; the quantity x_0 is determined as the nontrivial solution of the equation $f_1(x) = 0$. Clearly $x_0 \to 1$ as $\Pi_e \to \Pi_1$ and $x_0 \to 0$ as $\Pi_e \to \Pi_2$. The velocity distribution of the medium for the case under consideration takes the form:

$$U(x) = \begin{cases} \int_1^x [f_1(x)]^{\frac{1}{n}} dx, & x_0 < x < 1, \\ \int_x^{x_0} |f_1(x)|^{\frac{1}{n}} dx + \int_1^{x_0} [f_1(x)]^{\frac{1}{n}} dx, & 0 < x < x_0. \end{cases} \quad (1.7)$$

Equations (1.5)-(1.7), governing the velocity distribution of the liquid in the channel, enable us to carry out two limiting transitions, $n \to 1$ and $\Pi_e \to 0$. The first of these corresponds to the transition to a Newtonian liquid [2]. In this case, from (1.5)-(1.7) we have

$$U(x) = \frac{1 - x^2}{2} + \frac{\Pi_e}{2\alpha \operatorname{tg} \alpha} \ln \frac{\cos \alpha x}{\cos \alpha}. \quad (1.8)$$

The second limiting transition corresponds to the gradient flow of a non-Newtonian liquid with a power-type rheological law in the absence of an external electric field. If in Eq. (1.5) Π_e tends to zero, we obtain the well-known velocity distribution [10]:

$$U(x) = \frac{n}{n+1}\left(1 - x^{\frac{n+1}{n}}\right), \quad 0 < x < 1 \quad (1.9)$$

Figure 1a,b shows the characteristic velocity distribution of a liquid with various rheological constants in a plane channel for EHD flow; the distribution is calculated from Eqs. (1.5)-(1.7) as a function of the parameter Π_e for a single value of the parameter $\alpha = \pi/4$ ($\Pi_1 = -2$, $\Pi_2 = -8/\pi$).

2. Flow of a Power-Law Liquid in a Plane Channel with No Pressure Head. Let us consider the nongradient flow of a liquid with a power-type rheological law between plane walls ($x = \pm a$), one of which (let us say $x = a$) moves under the action of a tangential shear stress τ_w constant in time, while the second remains stationary. We take

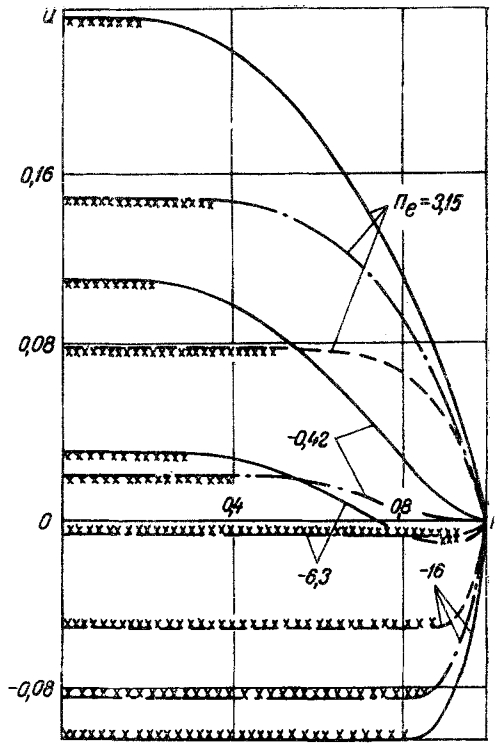

Fig. 3. Velocity distribution of a viscoplastic medium in a round tube (continuous curves, s = 0.1; broken curves, 0.5; dot-dash curves, 0.2).

the direction of the external field as coinciding with the positive direction of the longitudinal axis of the channel. As characteristic quantities we choose the half-width of the channel a and the velocity $V_0 = (q_0E_0/\eta)^{1/n}$. In order to make the conditions specific, let us say that q_0, $E_0 > 0$. We write down the equation of motion of the medium in dimensionless form

$$(|U'|^{n-1}U')' + \rho(x) = 0, \tag{2.1}$$

where the function $\rho = \rho(x)$ is defined by Eq. (1.1). The system of boundary conditions clearly takes the form

$$|U'(1)|^{n-1}U'(1) = \tau, \tag{2.2}$$

$$U(-1) = 0, \tag{2.3}$$

where $\tau = \tau_W/(q_0E_0)$ is the dimensionless tangential shear stress at the upper plate. Integrating Eq. (2.1) with respect to the transverse coordinate, allowing for the boundary condition (2.2), it is easy to derive

$$|U'(x)|^{n-1}U'(x) = f_2(x) = -\tau - \frac{1}{2}\left[\frac{\operatorname{tg}\alpha x}{\operatorname{tg}\alpha} - 1\right]. \tag{2.4}$$

Analyzing Eq. (2.4), we note that the derivative $U'(x)$ changes sign at the point $x = x_0$, where x_0 is a root of the equation $f_2(x) = 0$. The values $x_0 = 1$ and $x_0 = -1$ naturally define the characteristic values of the parameter τ, namely $\tau_1 = 0$ and $\tau_2 = -1$, which enable us to distinguish the following flow conditions (modes of flow).

a) $\tau > \tau_1$. In the case under consideration $U'(x) > 0$ at all internal points of the channel; the velocity distribution of the liquid, on allowing for the condition of adhering to the lower wall (2.3), takes the form

$$U(x) = \int_{-1}^{x} [f_2(x)]^{\frac{1}{n}}\, dx. \tag{2.5}$$

b) $\tau < \tau_2$. Here $U'(x) < 0$ at each internal point of the channel; hence

$$U(x) = -\int_{-1}^{x} |f_2(x)|^{\frac{1}{n}}\, dx. \tag{2.6}$$

c) $\tau_2 < \tau < \tau_1$. For this mode of flow the velocity distribution of the medium takes the form

$$U(x) = \begin{cases} \int_{-1}^{x} [f_2(x)]^{\frac{1}{n}}\, dx, & -1 < x < x_0, \\ \int_{-1}^{x_0} [f_2(x)]^{\frac{1}{n}}\, dx - \int_{x_0}^{x} |f_2(x)|^{\frac{1}{n}}\, dx, & x_0 < x < 1. \end{cases} \tag{2.7}$$

3. Gradient Flow of a Shvedov-Bingham Plastic in an Axisymmetrical Channel with a Dielectric Wall. Let us consider the one-dimensional flow of a medium with a rheological law in the form (1) (having n = 1) passing through a round tube with a dielectric wall under the influence of a pressure gradient $P = -(\partial p/\partial z) > 0$ (constant in time) in the presence of a longitudinal electric field E_0 = const. The density of the electric-charge distribution $q_0 = 2\pi \int_0^a \rho(x)\, x\, dx$, we shall regard as known by virtue of the solution of the electrodynamic part of the problem [15]. As defining parameters of the problem we take the total charge per unit length of tube $e_0 = q_0/(2\pi\varepsilon\varepsilon_0 a)$, and the radius of the tube a. Under these conditions the dimensionless distributions of the three-dimensional electric-charge density $\rho(x)$ and the value of the induced electric field $e(x)$ take the form

$$\rho(x) = \frac{1}{\pi}\,\frac{1+\alpha}{[1+\alpha(1-x^2)]^2}, \qquad (3.1)$$

$$e(x) = x\,[1+\alpha(1-x^2)]^{-1},$$

where $\alpha = q_0 b/(8\pi\varepsilon\varepsilon_0 D)$ is a dimensionless coefficient.

Passing to the hydrodynamic part of the problem, we write the equation of motion of the Shvedov–Bingham plastic in a round tube, allowing for the axial symmetry of the flow, thus:

$$\frac{1}{x}\,\frac{d}{dx}(x\tau) = -1 - \Pi_e \rho(x). \qquad (3.2)$$

Here τ is the tangential shear stress referred to the quantity Pa; $\Pi_e = q_0 E_0/Pa^2$ is the electric-interaction parameter (ratio of the electric-to-pressure forces). Integrating Eq. (3.2), while allowing for the flow symmetry $\tau(0) = 0$, we may write the distribution of the tangential shear stresses in the round tube as follows:

$$\tau(x) = -\frac{x}{2} - \Pi_e e(x). \qquad (3.3)$$

The distribution of the tangential shear stresses for $\alpha = 1$ is shown in Fig. 2 as a function of the electric-interaction parameter, in accordance with Eq. (3.3).

An analytical study of Eq. (3.3) shows that for $\Pi_e > \Pi_1 = -{}^{1/2}[2(1+2\alpha)]$ and $\Pi_e < \Pi_2 = -(1+\alpha)/2$ the distribution of the tangential shear stresses has no extremum in the range $0 < x < 1$. On satisfying the system of inequalities $\Pi_2 < \Pi_e < \Pi_1$, Eq. (3.3) reaches an extremal value $\tau_* = \tau(x_*)$, where x_* is given by the equation

$$x_* = \alpha^{-\frac{1}{2}}\left[1+\alpha-\Pi_e-\sqrt{\Pi_e(\Pi_e-4(1+\alpha))}\,\right]^{1/2}.$$

For $\Pi_e = \Pi_0 = -{}^1/_2$ the tangential shear stress vanishes on the tube wall, while for $\Pi_e = \Pi_w$ where Π_w is defined as the root of the algebraical equation $|\tau(x_*)| = \tau(1)$ the modulus of the extreme stress equals the stress at the wall of the tube.

In order to construct the velocity profile of the medium in the tube we have to find a solution to the equation defining the position of the interface between the zones of viscous flow and quasisolid motion $|\tau(x)| = s$, where $s = \tau_0/Pa$ is a dimensionless plasticity parameter; in the zones of viscous flow we use the rheological law (1) with n = 1, written in dimensionless form:

$$\tau = s\left|\frac{dU}{dx}\right|^{-1}\frac{dU}{dx} + \frac{dU}{dx}, \quad |\tau| > s. \qquad (3.4)$$

In Eq. (3.4) we take Pa^2/η as characteristic velocity. In integrating Eq. (3.4) we must allow for the condition that the medium should adhere to the tube wall, viz., $U(1) = 0$, and also the continuity condition for the velocity of the medium on passing through the interzone boundary.

It is convenient to analyze the possible modes of flow as functions of the electrical-interaction parameter.

a) $\Pi_e > \Pi_1$. If $s < {}^1/_2 + \Pi_e$, there is a unique solution to the equation $|\tau(x)| = s$ which we denote by x_0. In the mode under consideration viscous flow occurs in the boundary region $x_0 < x < 1$, the velocity being

$$U(x) = \frac{1-x^2}{4} + \frac{\Pi_e}{2\alpha}\ln[1+\alpha(1-x^2)] - s(1-x). \tag{3.5}$$

In the central flow zone $0 < x < x_0$ the medium moves as a single whole at a velocity $U_0 = U(x_0)$. If $s > {}^1/_2 + \Pi_e$, no flow of the medium takes place in the tube at all (the channel is blocked).

b) $\Pi_0 < \Pi_e < \Pi_1$. If $s < |\tau_*|$, the interzone boundaries of the flow are $x = x_{01}$ and $x = x_{02}$; the central zone $0 < x < x_{01}$ moves as a single whole, in the zone $x_{01} < x < x_{02}$ viscous flow takes place, while in the boundary zone $x_{02} < x < 1$ the system is at rest. The velocity distribution of the medium here takes the form ($|\tau(1)| < s < |\tau_*|$)

$$U(x) = \begin{cases} \dfrac{x_{02}^2 - x_{01}^2}{4} + \dfrac{\Pi_e}{2\alpha}\ln\dfrac{1+\alpha(1-x_{01}^2)}{1+\alpha(1-x_{02}^2)} + s(x_{01}-x_{02}), & 0 < x < x_{01}, \\ \dfrac{x_{02}^2 - x^2}{4} + \dfrac{\Pi_e}{2\alpha}\ln\dfrac{1+\alpha(1-x^2)}{1+\alpha(1-x_{02}^2)} + s(x-x_{02}), & x_{01} < x < x_{02}, \\ 0 & x_{02} < x < 1. \end{cases} \tag{3.6}$$

If $s > |\tau_*|$, the viscoplastic medium does not flow in the tube at all.

c) $\Pi_w < \Pi_e < \Pi_0$. In this range of Π_e the modulus of the extremal value of the tangential shear stress is no greater than the tangential shear stress at the tube wall. In this case for $s > |\tau_*|$ the viscoplastic medium is quiescent; for $-{}^1/_2 - \Pi_e < s < |\tau_*|$ the flow pattern is entirely analogous to the previous case (3.6), while for $0 < s < -{}^1/_2 - \Pi_e$ we have three interfaces x_{01}, x_{02}, x_{03}. There are two zones of quasi-solid motion in the tube, together with two zones of viscous flow; in the zone of viscous flow close to the wall $dU/dx > 0$ and in the central zone $dU/dx < 0$:

$$U(x) = \begin{cases} \dfrac{x_{02}^2 - x_{01}^2}{4} + \dfrac{\Pi_e}{2\alpha}\ln\dfrac{1+\alpha(1-x_{01}^2)}{1+\alpha(1-x_{02}^2)} + s(x_{01}-x_{02}) + \dfrac{1-x_{03}^2}{4} + \\ \quad + \dfrac{\Pi_e}{2\alpha}\ln[1+\alpha(1-x_{03}^2)] - s(x_{03}-1), \quad 0 < x < x_{01}, \\ \dfrac{x_{02}^2 - x^2}{4} + \dfrac{\Pi_e}{2\alpha}\ln\dfrac{1+\alpha(1-x^2)}{1+\alpha(1-x_{02}^2)} + s(x-x_{02}) + \dfrac{1-x_{03}^2}{4} + \\ \quad + \dfrac{\Pi_e}{2\alpha}\ln[1+\alpha(1-x_{03}^2)] - s(x_{03}-1), \quad x_{01} < x < x_{02}, \\ \dfrac{1-x_{03}^2}{4} + \dfrac{\Pi_e}{2\alpha}\ln[1+\alpha(1-x_{03}^2)] - s(x_{03}-1), \quad x_{02} < x < x_{03}, \\ \dfrac{1-x^2}{4} + \dfrac{\Pi_e}{2\alpha}\ln[1+\alpha(1-x^2)] - s(x-1), \quad x_{03} < x < 1. \end{cases} \tag{3.7}$$

d) $\Pi_2 < \Pi_e < \Pi_w$. In this mode of flow the tangential shear stress at the tube wall is greater than the absolute value of the extremal tangential shear stress. If $0 < s < |\tau_*|$, we have the flow described by the equations (3.7). If $|\tau_*| < s < -{}^1/_2 - \Pi_e$, the equation $|\tau(x)| = s$ has a unique root $x = x_0$, but in contrast to case a) the rate of flow of the medium is negative:

$$U(x)=\begin{cases}\dfrac{1-x_0^2}{4}+\dfrac{\Pi_e}{2\alpha}\ln[1+\alpha(1-x_0^2)]-s(x_0-1), & 0<x<x_0,\\ \dfrac{1-x^2}{4}+\dfrac{\Pi_e}{2\alpha}\ln[1+\alpha(1-x^2)]-s(x-1), & x_0<x<1.\end{cases} \quad (3.8)$$

For $s > -^1/_2 - \Pi_e$ the viscoplastic medium is quiescent.

e) $\Pi_e < \Pi_2$. In this case the velocity distribution of the flow obeys Eq. (3.8) for all $s < -\,^1/_2 - \Pi_e$.

Figure 3 shows the results of some calculations of the velocity profiles of a viscoplastic medium in a round tube for $\alpha = 9$ in relation to the electrical-interaction parameter Π_e for several values of the plasticity parameter s. For the sake of clarity the quasisolid zones are shown shaded in Fig. 3.

In conclusion, let us consider the physical interpretation of the foregoing solutions. First of all it should be noted that the three-dimensional electric-charge density in the plane and axisymmetrical channels (1.1) and (3.1) is not constant. The greatest electrical-charge density corresponds to the boundary region (next to the wall). Hence the ponderomotive forces arising in the liquid in the presence of an external electric field are greatest close to the walls of the channel.

For positive values of the parameter Π_e, when the ponderomotive forces coincide in direction with the pressure forces, the nonuniformity of the electric-charge distribution in the channel leads to a relative increase in the velocity of the liquid in the boundary regions for the case of EHD flow (by comparison with ordinary hydrodynamic flow). In other words, for $\Pi_e > 0$ an increase in the external electric field leads not only to an increase in the rate of liquid flow but also to the compression of the velocity profile.

For negative values of the electrohydrodynamic-interaction parameter $\Pi_e < 0$, when the ponderomotive forces are in the opposite direction to the pressure forces, an increase in the external electric field retards the liquid, this effect being greatest close to the walls of the channel. For a certain value of the parameter $\Pi_e < 0$ the ponderomotive forces exceed the pressure forces in magnitude in the region next to the wall. This leads to the appearance of a reverse flow of the medium in the boundary regions. On further increasing the external electric field with $\Pi_e < 0$, the velocity of the liquid becomes negative over the whole flow region.

NOTATION

τ is the tangential shear stress; τ_0 is the limiting tangential shear stress; U is the velocity of the liquid; x is the transverse coordinate; η and n are the rheological constants; a is the half-width of channel; E_0 is the external electric field; ρ is the three-dimensional electric-charge density; e is the induced electric field; q_0 is the charge per unit length of channel; ε is the dielectric constant of the medium; ε_0 is the electric constant; α is the dimensionless mobility coefficient; b is the mobility of the electric charges; D is the diffusion coefficient of the charged particles; P is the pressure gradient; U_0 and V_0 are the characteristic velocities; Π_e are the electrohydrodynamic interaction parameters; Π_0, Π_1, Π_2, Π_W are the characteristic values of the interaction parameters; x_{01}, x_{02}, x_{03} are the interzone boundaries of the various modes of flow.

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